MATHEMATICS

Department Website (http://www.math.sc.edu/)

Mathematics is a dynamic and collaborative field. Studying mathematics prepares students for future graduate studies not only in math, but in a wide variety of analytical disciplines such as computer science, physics and engineering. It can also help to prepare students for professional studies in fields like medicine and law. Fundamentally, a background in mathematics benefits those pursuing careers where problem-solving, high-level analysis, critical thinking and persistence are useful. Mathematical training teaches students how to read and communicate complex ideas as well as computational skills. Our department offers a major and minor in mathematics, with areas of emphasis in actuarial, education, applied or general mathematics.

The degree program offers two tracks: a program leading to the degree of Bachelor of Science in mathematics and a special five-year program leading to a Bachelor of Science degree and a Master of Science degree in mathematics. Students benefit from relatively small classes, including many upper-level sections for both Honors and advanced students. In addition, the department serves many of the disciplines within the university through course offerings which provide basic mathematical skills. A mathematics minor or double major complements many primary areas of study, particularly the sciences.

General Mathematics Courses

MATH 111 is a course in basic mathematics intended for students who plan to take MATH 122 or MATH 170 and who need more thorough development in algebraic methods.

MATH 111I is an intensive version of MATH 111. This course is intended for students who plan to take MATH 122 or MATH 170 and desire additional support—in the form of smaller classes and more contact hours—to develop the necessary algebraic skills.

MATH 112 is the basic trigonometry course for students who plan to take MATH 141 and have adequate preparation in algebra but need more thorough development in trigonometry. This course may not be used for mathematics credit in the College of Engineering and Computing.

MATH 115 is the basic precalculus course for students who plan to take MATH 141 and need more thorough development in algebra and trigonometry before entering MATH 141. This course may not be used for mathematics credit in the College of Engineering and Computing.

MATH 122 is intended for students in business, the social sciences, pharmacy, and other disciplines which require an introduction to mathematics credit in the College of Engineering and Computing. These courses are open to all students who satisfy the general requirements listed below.

MATH 141, MATH 142, MATH 241 constitute the normal calculus sequence for students in the College of Arts and Sciences and the College of Engineering and Computing. These courses are open to all students who satisfy the general requirements listed below.

MATH 170 is a basic course in finite mathematics. It may be used to satisfy the University's core requirements and is open to all interested students who satisfy the general requirements listed below.

Freshman Placement in Mathematics

MATH 111: Qualification through placement.

MATH 111I: Qualification through placement.

MATH 112: Qualification through placement or credit for MATH 111, either by successful completion of the course with a grade of C or better, transfer credit from another university, or successful completion of the test in MATH 111, available from the testing service.

MATH 115: Qualification through placement.

MATH 122: Qualification through placement or credit for MATH 111, either by successful completion of the course with a grade of C or better, transfer credit from another university, or successful completion of the test in MATH 111, available from the testing service.

MATH 141: Qualification through placement or credit for MATH 112 or MATH 115, either by successful completion of the course with a grade of C or better, transfer credit from another university, or successful completion of the test in MATH 115, available from the testing service.

Students who do not qualify for MATH 141 under paragraph 1 are strongly encouraged to try to obtain credit for MATH 115 either by taking the course or the examination during the summer preceding their first fall semester.

MATH 170: Qualification through placement or credit for MATH 111 or MATH 115, either by successful completion of the course with a grade of C or better, transfer credit from another university, or successful completion of the test in MATH 111 or MATH 115 which is available from the testing service.

Incoming students who wish to obtain bypass credit for certain mathematics courses may do so as follows:

MATH 111: CLEP Subject Examination titled “College Algebra” available from the testing service.

MATH 112: CLEP Subject Examination titled “Trigonometry” available from the testing service.

MATH 115: CLEP Subject Examination titled “College Algebra-Trigonometry” available from the testing service.

MATH 141: CLEP Subject Examination titled “Calculus with Analytic Geometry” available from the testing service.

Advanced Placement Test in Mathematics: The Advanced Placement Mathematics tests may be used to gain credit and advanced placement in calculus. Information is available from the testing service.

Programs

- Mathematical Biology Minor (https://academicbulletins.sc.edu/undergraduate/arts-sciences/mathematics/mathematical-biology-minor/)
- Mathematics Minor (https://academicbulletins.sc.edu/undergraduate/arts-sciences/mathematics/mathematics-minor/)
- Mathematics, B.S. (https://academicbulletins.sc.edu/undergraduate/arts-sciences/mathematics/mathematics-bs/)
Courses

**MATH 111 - Basic College Mathematics** *(3 Credits)*
Basic college algebra; linear and quadratic equations, inequalities, functions and graphs of functions, exponential and logarithm functions, systems of equations.
**Prerequisites:** placement through the Math Assessment of Prerequisites (MAP).

**MATH 111I - Intensive Basic College Mathematics** *(4 Credits)*
An intensive treatment of the topics covered in MATH 111.
**Prerequisites:** placement through the Math Assessment of Prerequisites (MAP).

**MATH 112 - Trigonometry** *(2 Credits)*
Topics in trigonometry specifically needed for MATH 141, MATH 142, MATH 241. Circular functions, analytic trigonometry, applications of trigonometry. Credit may not be received for both MATH 112 and MATH 115.
**Prerequisites:** C or better in MATH 111 or MATH 111I, or placement through the Math Assessment of Prerequisites (MAP).

**MATH 115 - Precalculus Mathematics** *(4 Credits)*
Topics in algebra and trigonometry specifically needed for MATH 141, MATH 142, MATH 241. Subsets of the real line, absolute value; polynomial, rational, inverse, logarithmic, exponential functions; circular functions; analytic trigonometry.
**Prerequisites:** C or better in MATH 111 or MATH 111I, or placement through the Math Assessment of Prerequisites (MAP).

**MATH 116 - Brief Precalculus Mathematics** *(2 Credits)*
Essential algebra and trigonometry topics for Calculus, including working with equations that involve polynomials, rational functions, exponential and logarithmic functions, and trigonometric and inverse trigonometric functions. Intended for students with prior experience in Precalculus, but not ready for MATH 141.
**Prerequisites:** C or better in MATH 112 or MATH 115, or placement through the Math Assessment of Prerequisites (MAP).

**MATH 122 - Calculus for Business Administration and Social Sciences** *(3 Credits)*
Derivatives and integrals of elementary algebraic, exponential, and logarithmic functions. Maxima, minima, rate of change, motion, work, area under a curve, and volume.
**Prerequisites:** C or better in MATH 111, MATH 111I or MATH 115 or placement through the Math Assessment of Prerequisites (MAP).

**Carolina Core:** ARP

**MATH 141 - Calculus I** *(4 Credits)*
Functions, limits, derivatives, introduction to integrals, the Fundamental Theorem of Calculus, applications of derivatives and integrals. Four classroom hours and one laboratory hour per week.
**Prerequisites:** C or better in MATH 112, MATH 115, or MATH 116, or placement through the Math Assessment of Prerequisites (MAP).

**Carolina Core:** ARP

**MATH 142 - Calculus II** *(4 Credits)*
Methods of integration, sequences and series, approximations. Four classroom hours and one laboratory hour per week.
**Prerequisites:** C or better in MATH 141.

**Carolina Core:** ARP

**MATH 151 - Calculus Workshop I** *(1 Credit)*
Small study group practice in applications of calculus. For elective credit only.
**Corequisite:** MATH 141.

**MATH 152 - Calculus Workshop II** *(1 Credit)*
MATH 152 is part of a pilot program at UofSC to help at-risk students in Calculus I & II. It is for elective credit only. The class will consist of small study group practice in applications of calculus.
**Corequisite:** MATH 142

**MATH 170 - Finite Mathematics** *(3 Credits)*
Elementary matrix theory; systems of linear equations; permutations and combinations; probability and Markov chains; linear programming and game theory.
**Prerequisites:** C or better in MATH 111 or MATH 111I or MATH 122, or placement through the Math Assessment of Prerequisites (MAP).

**Carolina Core:** ARP

**MATH 172 - Mathematical Modeling for the Life Sciences** *(3 Credits)*
Biological modeling with differential and difference equations; techniques of model modifications; analytic, numerical, and graphical solution methods; equilibria, stability, and long-term system behavior; geometric series; vectors, matrices, eigenvalues, and eigenvectors. Applications principally to population dynamics and compartment models.
**Prerequisites:** C or better in MATH 122 or MATH 141.

**Carolina Core:** ARP

**MATH 174 - Discrete Mathematics for Computer Science** *(3 Credits)*
Logic, number theory, sequences, series, recursion, mathematical induction, set theory, enumeration, functions, relations, graphs and trees. Connections to computers and to programming are emphasized when possible.
**Prerequisites:** C or better in MATH 112 or above, or placement through the Math Assessment of Prerequisites (MAP).

**Carolina Core:** ARP

**MATH 198 - Introduction to Careers and Research in the Mathematical Sciences** *(1 Credit)*
An overview of different areas of mathematical research and career opportunities for mathematics majors. Pass/fail only.
**Prerequisites:** C or better in MATH 141.

**Graduation with Leadership Distinction:** GLD: Research

**MATH 221 - Basic Concepts of Elementary Mathematics I** *(3 Credits)*
The meaning of number, fundamental operations of arithmetic, the structure of the real number system and its subsystems, elementary number theory. Open only to students in elementary or early childhood teacher certification.
**Prerequisites:** C or better in MATH 111 or MATH 111I or placement through the Math Assessment of Prerequisites (MAP).

**MATH 222 - Basic Concepts of Elementary Mathematics II** *(3 Credits)*
Informal geometry and basic concepts of algebra. Open only to students in elementary or early childhood teacher certification.
**Prerequisites:** C or better in MATH 221.
MATH 241 - Vector Calculus (3 Credits)
Vector algebra, geometry of three-dimensional space; lines, planes, and curves in space; polar, cylindrical, and spherical coordinate systems; partial differentiation, max-min theory; multiple and iterated integration, line integrals, and Green's theorem in the plane.
Prerequisites: C or better in MATH 142.

MATH 242 - Elementary Differential Equations (3 Credits)
Ordinary differential equations of first order, higher order linear equations, Laplace transform methods, series methods; numerical solution of differential equations. Applications to physical sciences and engineering.
Prerequisites: C or better in MATH 142.

MATH 300 - Transition to Advanced Mathematics (3 Credits)
Rigor of mathematical thinking and proof writing via logic, sets, and functions. Intended to bridge the gap between lower-level (computational-based) and upper-level (proof-based) mathematics courses.
Prerequisites: C or better in MATH 142.

MATH 328 - Mathematical Concepts for Data Analytics (3 Credits)
Prerequisites: C or better in MATH 122 or MATH 141; C or better in MATH 170.
Prerequisite or Corequisite: C or better in STAT 301 or STAT 516.

MATH 344 - Applied Linear Algebra (3 Credits)
General solutions of systems of linear equations, vector spaces and subspaces, linear transformations, determinants, orthogonality, characteristic polynomials, eigenvalues and eigenvectors, singular value decomposition, and generalized inverse. MATH 344L is an optional laboratory course where additional applications will be discussed.
Prerequisites: C or better in MATH 142.

MATH 344L - Applied Linear Algebra Lab (1 Credit)
Computer based applications of linear algebra for science and engineering students. Topics include numerical analysis of matrices, direct and indirect methods for solving linear systems, and least squares method (regression).
Prerequisite or Corequisite: C or better or concurrent enrollment in MATH 344 or MATH 544.

MATH 374 - Discrete Structures (3 Credits)
Propositional and predicate logic; proof techniques; recursion and recurrence relations; sets, combinatorics, and probability; functions, relations, and matrices; algebraic structures.
Prerequisites: C or better in both MATH 142 and either CSCE 106 or CSCE 146.

MATH 399 - Independent Study (3-9 Credits)
Contract approved by instructor, advisor, and department chair is required for undergraduate students.
Graduation with Leadership Distinction: GLD: Research

MATH 401 - Conceptual History of Mathematics (3 Credits)
Topics from the history of mathematics emphasizing the 17th century to the present. Various mathematical concepts are discussed and their development traced. For elective or Group II credit only.
Prerequisites: C or better in MATH 122, or MATH 141.

MATH 490 - Mathematics Internship (1-3 Credits)
Academic counterpart to a professional work experience in which mathematics plays a central role. Introduction to the uses of problem formulation and problem solving in a working environment. Introduction to career possibilities for a student trained in mathematics. Restricted to MATH major with 3.0 or better GPA and completion of at least 60 credits.
Prerequisites: C or better in MATH 241, MATH 300 and at least one 500 level MATH course; CSCE 145 or CSCE 106 and one of the following STAT courses STAT 509, STAT 512, STAT 515.

MATH 499 - Undergraduate Research (1-3 Credits)
Research on a specific mathematical subject area. The specific content of the research project must be outlined in a proposal that must be approved by the instructor and the Undergraduate Director. Intended for students pursuing the B.S. in Mathematics with Distinction. Pass-Fail grading only.
Graduation with Leadership Distinction: GLD: Research

MATH 511 - Probability (3 Credits)
Probability and independence; discrete and continuous random variables; joint, marginal, and conditional densities, moment generating functions; laws of large numbers; binomial, Poisson, gamma, univariate, and bivariate normal distributions.
Prerequisite or Corequisite: C or better in MATH 241.

Cross-listed course: STAT 511

MATH 514 - Financial Mathematics I (3 Credits)
Prerequisites: C or better in MATH 241.

Cross-listed course: STAT 522

MATH 515 - Financial Mathematics II (3 Credits)
Prerequisites: C or better in MATH 514 or STAT 522.

Cross-listed course: STAT 523

MATH 520 - Ordinary Differential Equations (3 Credits)
Differential equations of the first order, linear systems of ordinary differential equations, elementary qualitative properties of nonlinear systems.
Prerequisites: C or better in MATH 344 or MATH 544.
MATH 521 - Boundary Value Problems and Partial Differential Equations (3 Credits)
Laplace transforms, two-point boundary value problems and Green's functions, boundary value problems in partial differential equations, eigenfunction expansions and separation of variables, transform methods for solving PDE's, Green's functions for PDE's, and the method of characteristics.
Prerequisites: C or better in MATH 520 or in both MATH 241 and MATH 242.

MATH 522 - Wavelets (3 Credits)
Basic principles and methods of Fourier transforms, wavelets, and multiresolution analysis; applications to differential equations, data compression, and signal and image processing; development of numerical algorithms. Computer implementation.
Prerequisites: C or better in MATH 344 or MATH 544.

MATH 523 - Mathematical Modeling of Population Biology (3 Credits)
Applications of differential and difference equations and linear algebra modeling the dynamics of populations, with emphasis on stability and oscillation. Critical analysis of current publications with computer simulation of models.
Prerequisites: C or better in MATH 142, BIOL 301, or MSCI 311 recommended.

MATH 524 - Nonlinear Optimization (3 Credits)
Descent methods, conjugate direction methods, and Quasi-Newton algorithms for unconstrained optimization; globally convergent hybrid algorithm; primal, penalty, and barrier methods for constrained optimization. Computer implementation of algorithms.
Prerequisites: C or better in MATH 241 and one of MATH 344 or MATH 544.

MATH 525 - Mathematical Game Theory (3 Credits)
Two-person zero-sum games, minimax theorem, utility theory, n-person games, market games, stability.
Prerequisites: C or better in MATH 300 and in either of MATH 344 or MATH 544.

MATH 526 - Numerical Linear Algebra (4 Credits)
Matrix algebra, Gauss elimination, iterative methods; overdetermined systems and least squares; eigenvalues, eigenvectors; numerical software. Computer implementation. Credit may not be received for both MATH 526 and MATH 544. Three lectures and one laboratory hour per week.
Prerequisites: C or better in MATH 142.

MATH 527 - Numerical Analysis (3 Credits)
Interpolation and approximation of functions; solution of algebraic equations; numerical differentiation and integration; numerical solutions of ordinary differential equations and boundary value problems; computer implementation of algorithms.
Prerequisites: C or better in MATH 520 or in both MATH 242 and MATH 344.

Cross-listed course: CSCE 561

MATH 528 - Mathematical Foundation of Data Science and Machine Learning (3 Credits)
Unconstrained and constrained optimization, gradient descent methods for numerical optimization, supervised and unsupervised learning, various reduced order methods, sampling and inference, Monte Carlo methods, deep neural networks.
Prerequisites: C or better in MATH 344 or MATH 544.

MATH 529 - Introduction to Deep Neural Networks (3 Credits)
Prerequisites: C or better in MATH 328, MATH 344, or MATH 544.

MATH 531 - Foundations of Geometry (3 Credits)
The study of geometry as a logical system based upon postulates and undefined terms. The fundamental concepts and relations of Euclidean geometry developed rigorously on the basis of a set of postulates. Some topics from non-Euclidean geometry.
Prerequisites: C or better in MATH 300.

MATH 532 - Modern Geometry (3 Credits)
Projective geometry, theorem of Desargues, conics, transformation theory, affine geometry, Euclidean geometry, non-Euclidean geometries, and topology.
Prerequisites: C or better in MATH 300.

MATH 533 - Elementary Geometric Topology (3 Credits)
Topology of the line, plane, and space, Jordan curve theorem, Brouwer fixed point theorem, Euler characteristic of polyhedra, orientable and non-orientable surfaces, classification of surfaces, network topology.
Prerequisites: C or better in MATH 241 and MATH 300.

MATH 534 - Elements of General Topology (3 Credits)
Elementary properties of sets, functions, spaces, maps, separation axioms, compactness, completeness, convergence, connectedness, path connectedness, embedding and extension theorems, metric spaces, and compactification.
Prerequisites: C or better in MATH 241 and MATH 300.

MATH 540 - Modern Applied Algebra (3 Credits)
Finite structures useful in applied areas. Binary relations, Boolean algebras, applications to optimization, and realization of finite state machines.
Prerequisites: MATH 300.

MATH 541 - Algebraic Coding Theory (3 Credits)
Error-correcting codes, polynomial rings, cyclic codes, finite fields, BCH codes.
Prerequisites: C or better in MATH 300 and in either of MATH 344 or MATH 544.

MATH 544 - Linear Algebra (3 Credits)
Vectors, vector spaces, and subspaces; geometry of finite dimensional Euclidean space; linear transformations; eigenvalues and eigenvectors; diagonalization. Throughout there will be an emphasis on theoretical concepts, logic, and methods. MATH 544L is an optional laboratory course where additional applications will be discussed.
Prerequisites: C or better in MATH 241 and MATH 300.

MATH 546 - Algebraic Structures I (3 Credits)
Permutation groups; abstract groups; introduction to algebraic structures through study of subgroups, quotient groups, homomorphisms, isomorphisms, direct product; decompositions; introduction to rings and fields.
Prerequisites: C or better in MATH 300 and 544.
MATH 547 - Algebraic Structures II (3 Credits)
Rings, ideals, polynomial rings, unique factorization domains; structure of finite groups; topics from: fields, field extensions, Euclidean constructions, modules over principal ideal domains (canonical forms).
Prerequisites: C or better in MATH 546.

MATH 548 - Geometry, Algebra, and Algorithms (3 Credits)
Polynomials and affine space, Grobner bases, elimination theory, varieties, and computer algebra systems.
Prerequisites: C or better in MATH 300 and in one of MATH 344 or MATH 544.

MATH 550 - Vector Analysis (3 Credits)
Vector fields, line and path integrals, orientation and parametrization of lines and surfaces, change of variables and Jacobians, oriented surface integrals, theorems of Green, Gauss, and Stokes; introduction to tensor analysis.
Prerequisites: C or better in MATH 241.

MATH 551 - Introduction to Differential Geometry (3 Credits)
Parametrized curves, regular curves and surfaces, change of parameters, tangent planes, the differential of a map, the Gauss map, first and second fundamental forms, vector fields, geodesics, and the exponential map.
Prerequisites: C or better in MATH 241 and MATH 300.

MATH 552 - Applied Complex Variables (3 Credits)
Complex integration, calculus of residues, conformal mapping, Taylor and Laurent Series expansions, applications.
Prerequisites: C or better in MATH 241.

MATH 554 - Analysis I (3 Credits)
Least upper bound axiom, the real numbers, compactness, sequences, continuity, uniform continuity, differentiation, Riemann integral and fundamental theorem of calculus.
Prerequisites: C or better in MATH 241 and two 500-level classes requiring MATH 300: MATH 525, MATH 531, MATH 532, MATH 533, MATH 534, MATH 540, MATH 541, MATH 544, MATH 544, MATH 548, MATH 551, MATH 561, MATH 570, MATH 574, MATH 575, or MATH 580.

MATH 555 - Analysis II (3 Credits)
Riemann-Stieltjes integral, infinite series, sequences and series of functions, uniform convergence, Weierstrass approximation theorem, selected topics from Fourier series or Lebesgue integration.
Prerequisites: C or better in MATH 554.

MATH 561 - Introduction to Mathematical Logic (3 Credits)
Syntax and semantics of formal languages; sentential logic, proofs in first order logic; Godel's completeness theorem; compactness theorem and applications; cardinals and ordinals; the Lowenheim-Skolem-Tarski theorem; Beth's definability theorem; effectively computable functions; Godel's incompleteness theorem; undecidable theories.
Prerequisites: C or better in MATH 300.

MATH 562 - Theory of Computation (3 Credits)
Basic theoretical principles of computing as modeled by formal languages and automata; computability and computational complexity.
Prerequisites: C or better in CSCE 350 or MATH 300.

MATH 570 - Discrete Optimization (3 Credits)
Discrete mathematical models. Applications to such problems as resource allocation and transportation. Topics include linear programming, integer programming, network analysis, and dynamic programming.
Prerequisites: C or better in MATH 300 and in one of MATH 544 or MATH 344.

MATH 572 - Mathematical Foundation of Network Science (3 Credits)
Graphs and probability, Web graphs, random graphs, models for complex graphs, graph searching algorithms, eigenvalues, PageRank.
Prerequisites: C or better in MATH 374 or MATH 574; C or better in MATH 344 or MATH 544.

MATH 574 - Discrete Mathematics I (3 Credits)
Mathematical models; mathematical reasoning; enumeration; induction and recursion; tree structures; networks and graphs; analysis of algorithms.
Prerequisites: C or better in MATH 300.

MATH 575 - Discrete Mathematics II (3 Credits)
A continuation of MATH 574. Inversion formulas; Polya counting; combinatorial designs; minimax theorems; probabilistic methods; Ramsey theory; other topics.
Prerequisites: C or better in MATH 574.

MATH 576 - Combinatorial Game Theory (3 Credits)
Winning in certain combinatorial games such as Nim, Hackenbush, and Domineering. Equalities and inequalities among games, Sprague-Grundy theory of impartial games, games which are numbers.
Prerequisites: C or better in MATH 300 or MATH 374.

MATH 580 - Elementary Number Theory (3 Credits)
Divisibility, primes, congruences, quadratic residues, numerical functions. Diophantine equations.
Prerequisites: C or better in MATH 300.

MATH 587 - Introduction to Cryptography (3 Credits)
Design of secret codes for secure communication, including encryption and integrity verification: ciphers, cryptographic hashing, and public key cryptosystems such as RSA. Mathematical principles underlying encryption. Code-breaking techniques. Cryptographic protocols.
Prerequisites: C or better in CSCE 145 or MATH 241, and at least one of CSCE 355, MATH 300, or MATH 374.

Cross-listed course: CSCE 557

MATH 590 - Undergraduate Seminar (1-3 Credits)
A review of literature in specific subject areas involving student presentations. Content varies and will be announced in the Master Schedule of Classes by title. For undergraduate credit only.

MATH 599 - Topics in Mathematics (1-3 Credits)
Recent developments in pure and applied mathematics selected to meet current faculty and student interest.

MATH 602 - An Inductive Approach to Geometry (3 Credits)
This course is designed for middle-level pre-service mathematics teachers. This course covers geometric reasoning. Euclidean geometry, congruence, area, volume, similarity, symmetry, vectors, and transformations. Dynamic software will be utilized to explore geometry concepts. This course cannot be used for credit toward a major in mathematics.
Prerequisites: C or better in MATH 122 or MATH 141 or equivalent.
MATH 603 - Inquiry Approach to Algebra (3 Credits)
This course introduces basic concepts in number theory and modern algebra that provide the foundation for middle level arithmetic and algebra. Topics include: algebraic reasoning, patterns, inductive reasoning, deductive reasoning, arithmetic and algebra of integers, algebraic systems, algebraic modeling, and axiomatic mathematics. This course cannot be used for credit towards a major in mathematics.  
**Prerequisites:** C or higher in MATH 122 or MATH 141 or equivalent.

MATH 650 - AP Calculus for Teachers (3 Credits)
A thorough study of the topics to be presented in AP calculus, including limits of functions, differentiation, integration, infinite series, and applications. Not intended for degree programs in mathematics.  
**Prerequisites:** current secondary high school teacher certification in mathematics and a C or better in at least 6 hours of calculus.